Stochastic dynamic models for low count observations

(and forecasting from them)

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Overview

1. Poisson-binomial state-space model
2. Inference and forecasting
3. Forecasting with noisy observations
4. Forecasting with inference on parameters

S→I→R stochastic compartment model

Focus on Susceptible (S) - Infectious (I) - Recovered (R) model with Poisson transitions, i.e.

$$\overrightarrow{SI} \sim Po(\lambda_{\mathrm{S}\to\mathrm{I}}\mathrm{SI}/\mathrm{N}) \qquad \overrightarrow{IR} \sim Po(\lambda_{\mathrm{I}\to\mathrm{R}}\mathrm{I})$$

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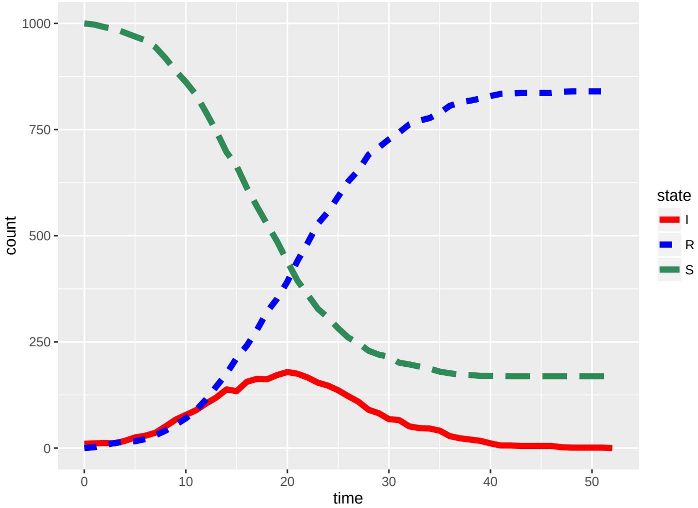
A more general stochastic dynamic modeling structure can be used to extended to geographical regions, subpopulations, etc.

SIR modeling simulations



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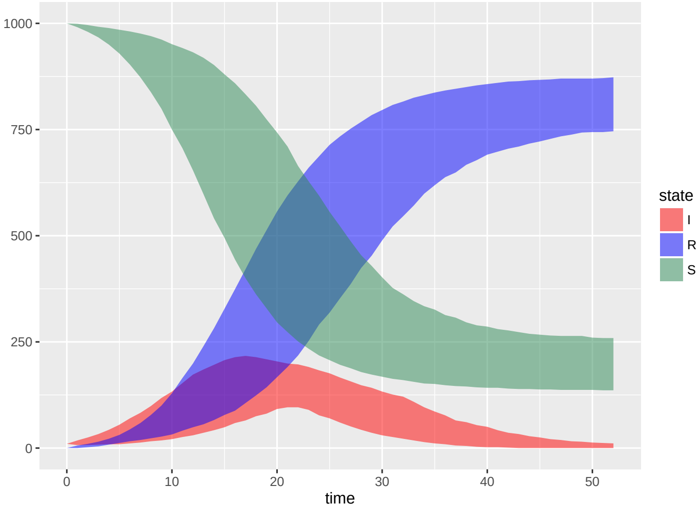

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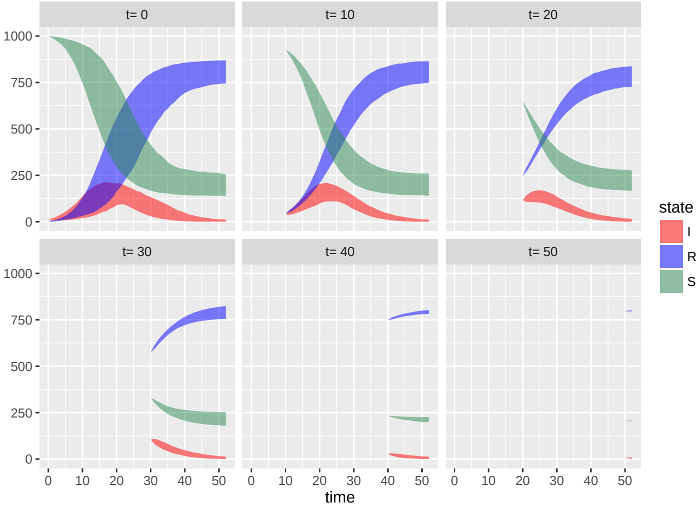
this distribution is estimated via Monte Carlo simulation.


t= 0
t= 10
t= 20
t= 30
t= 40
t= 50
1000
750
500
250
0
0 10 20 30 40 50
time
state
I
R
S

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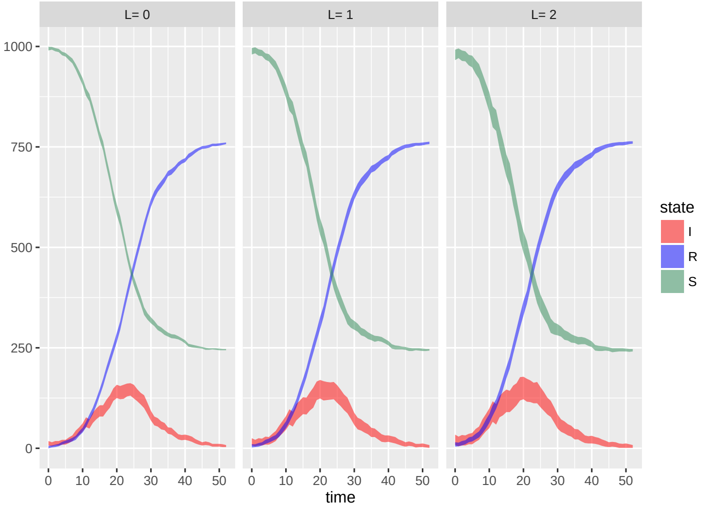
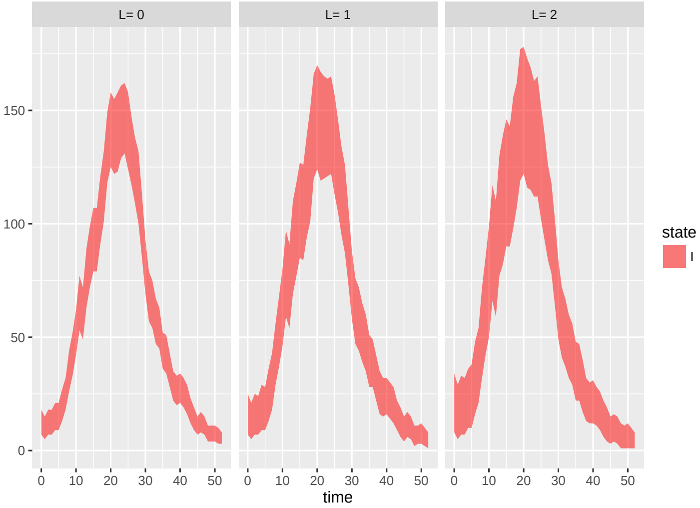
where

- $L = 0$ indicates up-to-date data
- $L = 1$ indicates one-week old data
- $L = 2$ indicates two-week old data


L= 0
L= 1
L= 2
1000
750
500
250
0
0 10 20 30 40 50
time
state
I
R
S


L= 0
L= 1
L= 2
150
100
50
0
0
10
20
30
40
50
time
state
I

Forecasting with noisy observations

Suppose, we know the transition rates (λ) and the observation probabilities (θ), but we only observe a noisy version of the state transitions, i.e. .

$$Y_{\mathsf{S}\to\mathsf{I}} \sim Bin(\overrightarrow{SI}, \theta_{\mathsf{S}\to\mathsf{I}}) \qquad Y_{\mathsf{I}\to\mathsf{R}} \sim Bin(\overrightarrow{IR}, \theta_{\mathsf{I}\to\mathsf{R}})$$

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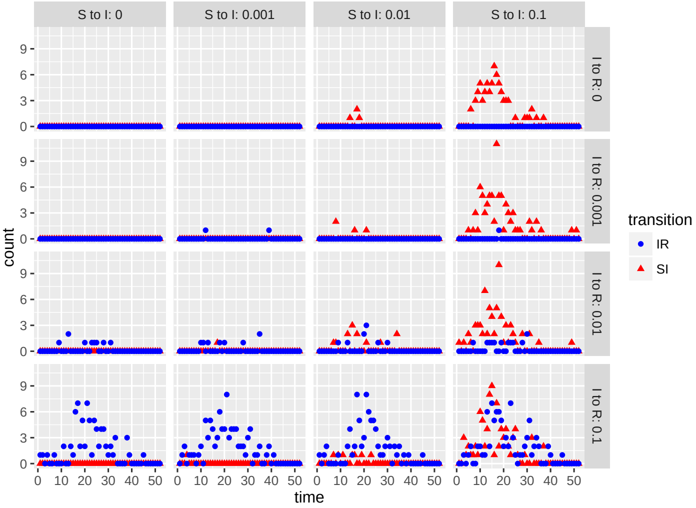
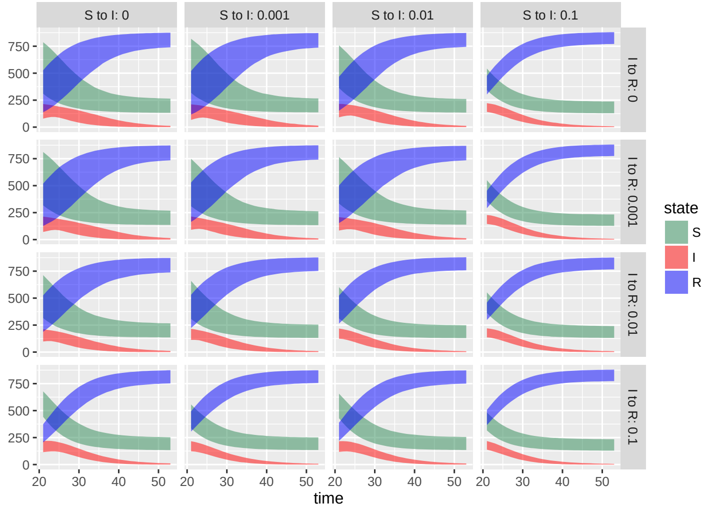
Now the forecast distribution we need is

$$p(X_{t+1:T}|\lambda, \theta, y_{0:t}) = \int p(X_{t+1:T}, \lambda, \theta | X_t) p(X_t|\lambda, \theta, y_{0:t}) dX_t.$$


S to I: 0
S to I: 0.001
S to I: 0.01
S to I: 0.1
I to R: 0
I to R: 0.001
I to R: 0.01
I to R: 0.1
count
time
transition
IR
SI

Noisily observed state



Forecasting with inference on parameters

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Prior distributions

In order to calculate (or approximate) the integral

$$\int \int \int p(X_{t+1:T}, \lambda, \theta | X_t) p(X_t, \lambda, \theta | y_{0:t}) d\lambda d\theta dX_t$$

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$$\begin{array}{ll} \theta_k & \stackrel{ind}{\sim} Be(n_\theta p_k, n_\theta[1-p_k]) \\ \lambda_k & \stackrel{ind}{\sim} Ga(n_\lambda c_k, n_\lambda) \\ X_0 & \sim Mult(N; z_1, \ldots, z_S) \end{array}$$

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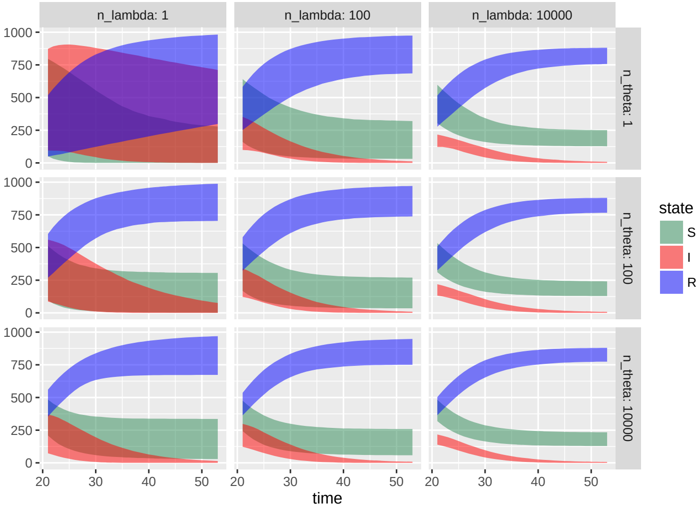
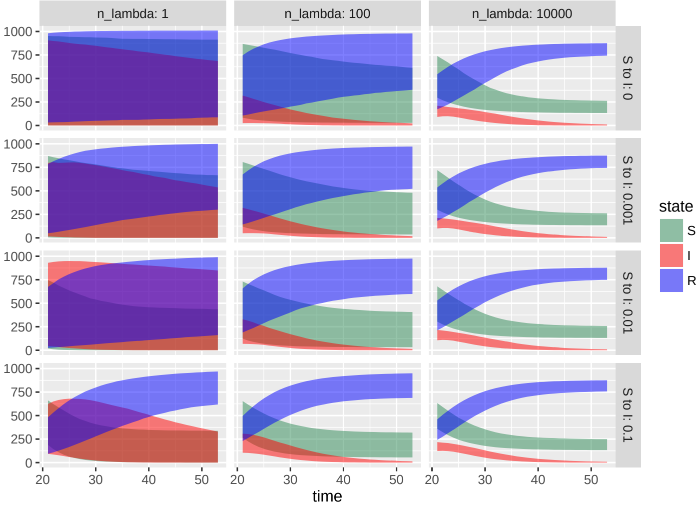
We can control how informative the priors are with n_θ and n_λ.

Informative priors



Balance priors and data



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Then, we can discuss how to assign resources depending on the costs associated with each impact above.