Sparse sensor placement optimization for classification (SSPOC)

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IDM, Disease Modeling Symposium
Given a fixed budget of sensors, where should they be placed to optimally inform decision-making?

sensor networks in biology

sensor networks for measurement and surveillance

image by Brad Dickerson
Relatively simple patterns often underly complex data.

Pixel space is larger than astronomical.
Compression and Compressive Sensing

10% random† measurements

reconstruct by solving for sparse representation

† subject to some specific constraints
Reconstruction by Compressive Sensing

To reconstruct:

\[
\text{minimize } \|s\|_1, \\
\text{such that } y = \Theta s.
\]

- Candès, Romberg & Tao, 2006.
- Donoho, 2006.

single pixel camera, reconstructions from http://dsp.rice.edu/cscamera
Why does $l_1$-minimization promote sparsity?

\[ |x|_2 = \sqrt{\sum_i |x_i|^2} \]

\[ |x|_1 = \sum_i |x_i| \]

\[ \|x\|_2 \]

\[ \|x\|_1 \]

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from Bryan & Leise, SIAM Review 2013
Simple Example: Beating Nyquist Sampling

\[ f(t) = \sin(73 \times 2\pi t) + \sin(531 \times 2\pi t) \]

Nyquist: 1062 samples/second
Compressed Sampling: 128 samples/second

Signal Reconstruction

Nyquist: 1062 samples/second
Compressed Sampling: 128 samples/second
Sparse sensor placement optimization for classification (SSPOC)
Acquire full measurements → Compress data → Reconstruct full data → Make decision

Acquire compressed measurements → Reconstruct full data → Make decision

Acquire very few, key measurements → Make decision
To solve for **sparse sensor locations**, 

\[ s = \operatorname{argmin}_{s'} \|s'\|_1, \quad \text{subject to} \quad \Psi^T r s' = w. \]

\( s \) is mostly zeros; the non-zero elements correspond to sensor locations, where we want to measure.

Image has \( n \) pixels

\( \Psi_r \) feature basis, \( n \times r \)

\( w \) decision vector, \( r \times 1 \)

\( S \) sparse sensors, \( n \times 1 \)

\( n \gg r \)

\[ \eta = (\Psi_r w)^T x \]

Measurements Features Classification

Full Measurements $\mathbb{R}^n \cdot x$

Random Subsamples $\mathbb{R}^p \cdot \tilde{x}$

Sparsely Sensed Measurements $\mathbb{R}^q \cdot \hat{x}$
Which person is in the picture?

ensemble
of sparse sensor locations

SSPOC on human faces

image

Yarbus, 1967.

human gaze
SRBCT cancer type:

What is the tumor type?

microarray dataset measured 2308 genes for 83 samples

28 sparse sensors:
0.95 ± 0.083 accuracy

microarray data from http://home.ccr.cancer.gov/oncology/oncogenomics/
Number of Genes Probed

Accuracy

Number of Genes Probed

Gene ID

RMS  EWS  NB  BL

What is the tumor type?

more measurements may not always be better!
Probabilistic reduced order model of dynamic regimes with sparse sensors

Kaiser et al., Cluster-based reduced-order modeling of a mixing layer, J Fluid Mechanics 2014.
**Verbal Autopsy:** given a budget of questions to ask, which ones are most informative of HIV status?

Karonga VA
from ALPHA dataset

with C. Calvert, S. Clark, T. McCormick, UW Depts. of Sociology & Statistics